

# **Measuring and Maximizing Resilience of Freight Transportation Networks**



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## **Executive Summary**

In assessing a network's potential performance given possible future disruptions, one must recognize the contributions of the network's inherent ability to cope with disruption via its topological and operational attributes and potential actions that can be taken in the immediate aftermath of such an event. Measurement and maximization of network resilience that accounts for both in the context of intermodal freight transport are addressed herein. That is, the problem of measuring a network's maximum resilience level and simultaneously determining the optimal set of preparedness and recovery actions necessary to achieve this level under budget and level-of-service constraints is formulated as a two-stage stochastic program. An exact methodology, employing the integer L-shaped method and Monte Carlo simulation, is proposed for its solution. Optimal allocation of a limited budget between preparedness and recovery activities is explored on an illustrative problem instance involving a network abstraction of a United States rail-based intermodal container network.

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# CHAPTER 1: INTRODUCTION AND MOTIVATION

Freight transportation infrastructure and related transport elements (trains, ships, planes and trucks) comprise a crucial lifeline for society. In the United States (U.S.), for example, an extensive freight transportation system, with a network of 4 million miles of roadway, nearly 140,000 miles of rail, approximately 25,000 miles of waterways, more than 350,000 intermodal terminals, almost 10,000 coastal and inland waterway facilities, and over 5,000 public-use airports (USDOT RITA BTS 2010), enables the expedient movement of raw materials, other resources, and end-products between suppliers, manufacturers, wholesalers, retailers and customers. Its expediency and efficiency are in large part due to an open, accessible design. This design, that supports mobility objectives, leaves the system vulnerable to malicious and random acts with the aim or unintended consequence of disrupting operations. Even minor disruptions can have effects that ripple through the network, resulting in major reductions in system efficiency with nation-wide or even global impact (as discussed in Miller Hooks et al., 2009).

This paper proposes a method for assessing and maximizing the resilience of an intermodal freight transport network. Resilience involves both the network's inherent ability to cope with disruption via its topological and operational attributes and potential actions that can be taken in the immediate aftermath of a disruption or disaster event. This conceptualization of resilience in terms of both inherent and adaptive components is discussed in (Rose, 2004) and was first quantified in (Chen and Miller-Hooks, forthcoming). See also (Nair et al., 2010) in which the concept was applied in a component-based application involving the intermodal port of Świnoujście in Poland.

Chen and Miller-Hooks (forthcoming) formulated the problem of measuring resilience,

defined as the expected system throughput given a fixed budget for recovery action and fixed system demand, in an intermodal freight transportation application. The problem was posed as a stochastic, mixed integer program. The program includes no first-stage variables. All decisions are taken once the outcome of the random disaster event is known. Thus, the problem can be decomposed into a set of independent, scenario-specific, deterministic (albeit NP-hard) problems and the focus of their solution approach is on the sampling methodology and exact solution of each independent deterministic problem that results for a given network state. They presented a solution framework employing Benders decomposition (Benders, 1962), column generation and Monte Carlo simulation. A secondary outcome from solving the mathematical program is the optimal set of recovery actions that can be taken to obtain the maximum attainable throughput for each potential network state.

This conceptualization of resilience that includes not only the network's inherent coping capacity, but also the potential impact of immediate recovery action within a limited budget is illustrated in Figure 1.

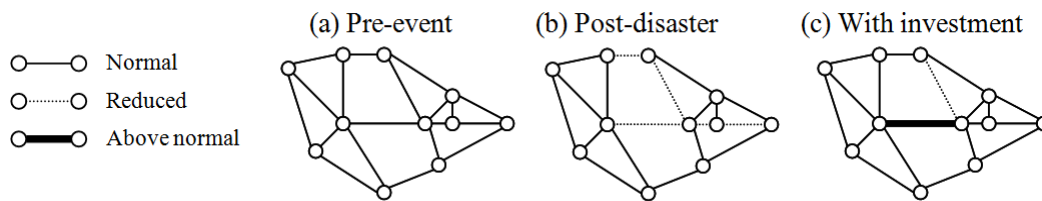


Figure 1. Illustrative example showing impact of recovery activities on system performance

As depicted in the figure, post-disaster arc capacities may be significantly reduced for affected arcs (Figure 1(b)), resulting in a poorly performing network. However, if recovery actions can be taken in the immediate aftermath of disaster to restore and even improve at least a subset of the affected arcs (Figure 1(c)), and such restoration can be accomplished

quickly and within an acceptable budget, one may view the network *a priori* as highly performing, i.e. resilient.

This paper expands on this conceptualization of resilience by incorporating preparedness decisions and capturing synergies between preparedness activities and recovery options. The concept of resilience as defined by Chen and Miller-Hooks was developed as a strategic tool. It permits the measurement of a network's resilience level given a set of possible, future network states and potential remedial actions. Remedial actions that may be taken pre-event (e.g. adding additional links to the network, ordering spare parts or backup equipment, prepositioning resources in anticipation of potential recovery activities, implementation of advanced technologies, training, and other pre-event actions that can reduce the time or budget required to complete potential recovery activities should they be required post-event) were not considered in their work. Thus, their formulation does not include pre-event, i.e. first-stage, decision variables and their approach does not incorporate decisions concerning actions that can be taken pre-disaster.

In determining resilience within this paper, a budget is available for both preparedness and recovery options. A two-stage stochastic, integer program is presented in which preparedness decisions are taken in the first stage and recovery actions are suggested for each disaster scenario in the second stage. The incorporation of first-stage decisions precludes the problem's decomposition into a set of independent, deterministic problems as was possible when only recovery options were considered (i.e. as in (Chen and Miller-Hooks, forthcoming)). An integer L-shaped method is proposed herein for solution of the resilience problem with preparedness options.

By incorporating risk mitigation through preparedness investment within a framework that accounts for the inherent network coping capacity along with immediate post-disaster

recovery options, the developed solution methodology will provide tactical support for improving pre-disaster preparedness and post-disaster response, thus, achieving an optimal balance between preparedness and recovery investment. While this paper focuses on intermodal freight transport networks, general concepts developed herein have wider applicability.

In the next chapter, related works in the literature are reviewed. In Chapter 3, the concept of resilience with preparedness options is defined. A two-stage stochastic integer program is presented for obtaining the allocation of funds to preparedness and recovery activities such that resilience, measured in terms of expected system throughput, is maximized. The formulation accounts for the inherent coping capacity of the network, along with the impact of cost-effective preparedness and recovery actions that can be taken to preserve or restore the system's ability to perform its intended function in a disaster's aftermath. The integer L-shaped method proposed for its solution is presented in Chapter 4. Solution of the program results in a measure of maximum resilience in terms of expected throughput for a given budget level, as well as the preparedness and post-disaster actions that are needed to achieve the maximum resilience value. Finally, in Chapter 5, these concepts and the effectiveness of the proposed solution methodology are illustrated on the Double-Stack Container Network (Morlok and Chang, 2004; Sun et al., 2006) under disaster scenarios involving a terrorist attack, flooding or earthquake. Results of the numerical study show the additional benefits in terms of increased resilience level that are derived from taking preparedness actions. In addition, the optimal allotment of the budget between preparedness and recovery options for the case study is investigated.

## CHAPTER 2: LITERATURE REVIEW

Numerous works in the literature address network vulnerability, reliability and flexibility. These concepts are not always well defined and their meaning often varies from one work to another. It is only in rare cases, however, that consideration is given to actions that can be taken in the immediate aftermath of the disaster to improve system performance. An overview of the concepts of vulnerability, reliability, flexibility and resilience in the literature is given in (Chen and Miller-Hooks, forthcoming). Prior to (Chen and Miller-Hooks, forthcoming) and (Nair et al., 2010), a few works have considered compatible notions. Havidán et al. (2006) developed qualitative measures of resilience related to business contingency planning that account for actions taken to mitigate event impact. Srinivasan (2002) and Chatterjee (2002) espouse the need for recovery planning in the context of intermodal freight systems. Srinivasan (2002) advocates for a comprehensive and quantitative vulnerability index that can account for recovery potential, but does not provide such an index. A number of additional works recognize the importance of having resilient systems. Bruneau et al. (2003) emphasize that resilient systems reduce the probability of failure and its consequences, as well as the time for recovery. Other works consider recovery actions in the wake of natural and human-induced disasters (Daryl (1998), Williams et al. (2000), and Juhl (1993)), but do not consider a network performance measurement.

Ip and Wang (2009) define network resilience as a function of the number of reliable paths between all node pairs. Such a concept is consistent with other notions of network reliability. Ta et al. (2009) developed a qualitative definition of resilience in the context of freight transport as a tool for visualizing disruption consequences. Their definition captures the interactions between managing organizations, the infrastructure, and its users.



Murray-Tuite (2006) also introduces a quantitative measure of resilience designed to consider network performance under disruption. The measure involves 10 dimensions: redundancy, diversity, efficiency, autonomous components, strength, collaboration, adaptability, mobility, safety and the ability to recover quickly. She considered both system optimal and user equilibrium traffic assignment modeling approaches and examined and compared the network performance based on the last four dimensions of the resilience measure.

Numerous works address disaster operations management, which can affect network performance in a disaster's aftermath. A review of many of these works can be found in (Altay and Green, 2006). Altay and Green (2006) categorized these works based on the phases of the disaster management lifecycle phases addressed or type of disaster considered. Examples of works addressing disaster operations management include (Feng and Wen, 2005; Kondaveti et al., 2009; Chen and Miller-Hooks, 2011). Feng and Wen (2005) propose a decision-making tool using bi-level stochastic programming for planning emergency response in disasters due to earthquake. At the upper level, the program seeks to maximize coverage of the network by emergency vehicles, while at the lower level, the program seeks control of vehicular traffic such that total post-disaster vehicular travel times are minimized. Kondaveti et al. (2009) studied optimal resource deployment in post-disaster emergency response. In a similar context, Chen and Miller-Hooks (in review) propose a multi-stage stochastic programming formulation and exact solution approach for the problem of optimally deploying urban search and rescue teams to disaster sites in post-disaster circumstances where a portion of the sites requiring assistance arrive dynamically over the decision horizon and demand levels at the sites and on-site service times are known only with uncertainty a priori. These works seek an optimal post-disaster allocation of resources and optimal recovery actions that can be taken in a disaster's immediate aftermath, but do not attempt to

assess network performance *a priori*.

In this paper, not only are short-term recovery options incorporated within a quantitative pre-disaster measure of resilience, but preparedness actions that can affect recovery capacity are considered. To address the preparedness phase, Rawls and Turnquist and (2010) developed an emergency response plan in which supplies are pre-positioned. Peeta et al. (2010) addressed pre-disaster planning as an investment problem with the aim of strengthening the highway network and enhancing its ability to cope with disaster. The objective of their work is to maximize post-disaster connectivity and minimize traversal costs. Huang et al. (2007) seek the optimal locations for fire stations and number of emergency vehicles to position at each station such that coverage of critical transportation infrastructure components is maximized in preparation for a major disaster. Zhu et al. (2010) determine optimal storage locations and capacities to meet post-disaster demand that is known in advance only with uncertainty.

Two additional works that consider preparedness in this context include (Liu et al., 2009 and Fan and Liu, 2010). These works examine the allocation of limited resources (i.e. a fixed budget) for retrofitting and repairing bridges within a region. It was assumed that if a bridge were retrofitted, it would withstand all disaster forces. They further assume that non-retrofitted bridges will be damaged and impassable in the event of disaster and all damaged bridges will require repair. They model the problem of optimal investment in either pre-disaster retrofitting or post-disaster repair as a two-stage stochastic program in which a fixed budget is available for retrofitting the bridges. The objective is to minimize a function of travel and repair costs. Liu et al. (2009) compute travel costs from total travel time incurred by all vehicles in the system. Fan and Liu (2009) compute travel costs assuming an user equilibrium is reached. In both works, travel times are based on conditions in the

immediate aftermath of the disaster. Liu et al. (2009) require that all demand be met in each disruption scenario, while Fan and Liu (2010) only penalize solutions containing unsatisfied demand. Neither work considers the impact of repair on system performance.

Liu et al. (2009)'s two-stage stochastic program involves a nonlinear recourse function in the second stage. They propose a modified L-shaped method to solve the problem using a convex and piecewise linear approximation of the second stage objective function. The formulation given in Fan and Liu (2010) can be categorized as a mathematical program with complementarity constraints due to the traffic flow equilibrium constraints. As the L-shaped method relies on an assumption of convexity, an alternative solution approach is required. Thus, they propose a progressive hedging (PH) method due to (Rockafellar and Wets (1991)) that decomposes the problem into multiple independent subproblems based on scenarios. Within this solution framework, the subproblems are further reformulated into a series of mixed integer nonlinear programs in which the complementarity constraints are relaxed. Each such program is solved using a commercial solver. One can view the techniques proposed herein as extending the capabilities proposed in Liu et al.'s work to include recovery actions in the second stage as proposed in Chen and Miller-Hooks (forthcoming), as well as retrofit and repair options with intermediate, as opposed to all-or-nothing, impact. This work also provides a network performance measurement tool, which was not the goal of these earlier works.

Stochastic programming has been applied in numerous arenas, including, for example, production management, financial modeling, logistics, energy, pollution control, and healthcare. See (Wallace and Ziemba, 2005) for a review of stochastic programming applications. This approach is particularly well suited to the problem posed herein, as generally two-stage stochastic programs can be seen as having a preparedness stage, i.e. a

stage in which decisions must be taken prior to the realization of a random event, and a recovery stage, referred to as recourse, in which changes to earlier decisions can be made to improve the solution once the random values are actualized.

It appears that no prior work in the literature provides a network resilience measurement tool incorporating both preparedness and post-disaster recovery actions, as well as the potential impact of those actions.

## CHAPTER 3: PROBLEM DEFINITION

In this chapter, the problem of measuring resilience given preparedness options is defined. To the extent possible, for consistency, notation and definitions presented in (Chen and Miller-Hooks, forthcoming) are used.

As in the previous work, network resilience is defined as the expected fraction of demand that can be satisfied post-disaster:

$$\alpha = E\left(\sum_{w \in W} d_w / \sum_{w \in W} D_w\right) = \left(1 / \sum_{w \in W} D_w\right) \cdot E\left(\sum_{w \in W} d_w\right) \quad (1)$$

where  $D_w$  is the original pre-disaster demand for O-D pair  $w$ .  $d_w$  is the post-disaster maximum demand that can be satisfied for O-D pair  $w$ . Demand that can be satisfied depends on the inherent coping capacity of the network and post-disaster recovery actions taken to restore or enhance network capacity. The network's inherent ability to cope with disaster can be enhanced through preparedness efforts. Moreover, new options for actions that can be taken in the immediate aftermath of disaster may exist if certain preparedness actions are taken. That is, preparedness actions can affect recovery capacity.

The problem of measuring resilience given preparedness options, referred to herein as Resilience with Preparedness Options (RPO), is formulated as a two-stage stochastic program. The first stage includes decisions on pre-disaster preparedness actions, actions that would be taken prior to disaster realization. The second stage, the recourse stage, involves the selection of post-disaster recovery actions to take in the aftermath of disruption, once the impact of the disaster on network performance, specifically on arc capacities and traversal times, is known.

A network representation of the intermodal system is exploited in the formulation and

solution framework. Let  $G=(N,A)$ , where  $N$  is the set of nodes and  $A$  is the set of links. To address intermodal movements within the system, a representation employing intermodal connections between modal links would ordinarily be employed. For simplicity, intermodal movements are considered only in recovery options, and thus, in the following problem definition, modal links are indistinguishable. Notations employed in the problem formulation are synopsisized as follows.

$W$	=	set of O-D pairs
$K_w$	=	set of paths $k$ connecting O-D pair $w$
$D_w$	=	original demand between O-D pair $w$
$R$	=	Set of available recovery actions
$b_{ar}$	=	cost of implementing recovery activity $r \in R$ on arc $a$
$P$	=	Set of available preparedness actions
$b_{ap}$	=	cost of implementing preparedness activity $p \in P$ on link $a$
$b_{ar}^p$	=	cost of implementing recovery activity $r$ on arc $a$ if preparedness action $p$ is taken
$B$	=	given budget
$c_a(\omega)$	=	post-disaster capacity of link $a$ for disruption scenario $\omega$
$\Delta c_{ap}$	=	augmented capacity of link $a$ given preparedness action $p$ is taken
$\Delta c_{ar}(\omega)$	=	augmented capacity of link $a$ due to implementing recovery activity $r$ for disruption scenario $\omega$
$t_a(\omega)$	=	Traversal time of link $a$ under disruption scenario $\omega$
$t_{ar}$	=	Traversal time of link $a$ if recovery activity $r$ is implemented
$q_{ar}$	=	Implementation time of recovery activity $r$ on link $a$
$q_{ar}^p$	=	Traversal time of link $a$ if related preparedness action $p$ and recovery action $r$ is implemented

- $Q_k^w(\omega)$  = Maximum implementation time of recovery actions on path  $k$  between O-D pair  $w$   
 $T_w^{\max}$  = Maximum allowed traversal between O-D pair  $w$   
 $\lambda$  = Preparedness-recovery action relationship matrix in which each element  $\lambda_{pr}$  is set to 1 if recovery action  $r$  is affected by preparedness action  $p$  and 0 otherwise.  
 $\delta_{ak}^w$  = path-link incidence (=1 if path  $k$  uses link  $a$  and =0 otherwise)

Decision variables

- $\beta_{ap}$  = binary variable indicating whether or not preparedness activity  $p$  is undertaken on link  $a$  (=1 if preparedness action  $p$  is taken on link  $a$  and =0 otherwise)  
 $y_k^w(\omega)$  = binary variable indicating whether or not shipments use path  $k$  (=1 if path  $k$  is used and =0 otherwise) between O-D pair  $w$   
 $f_k^w(\omega)$  = post-disaster flow of shipments along path  $k$  between O-D pair  $w$  under scenario  $\omega$   
 $\gamma_{ar}(\omega)$  = binary variable indicating whether or not recovery activity  $r$  is undertaken on link  $a$  in the aftermath of disruption scenario  $\omega$  (=1 if recovery action  $r$  is taken on link  $a$  and =0 otherwise)

Based on this notation, resilience with preparedness options (RPO) is formulated as the following two-stage stochastic program:

(RPO)

First stage:

$$\max E_{\tilde{\omega}}[Z(\omega)] \quad (2)$$

s.t.

$$\sum_p \beta_{ap} \leq 1, \forall a \in A \quad (3)$$

$$\beta_{ap} \in \{0,1\}, \quad \forall a \in A, p \in P \quad (4)$$

Second stage:

$$Z(\omega) = \max \sum_{w \in W} \sum_{k \in K_w} f_k^w(\omega) \quad (5)$$

s.t.

$$\sum_{k \in K_w} f_k^w(\omega) \leq D_w, \quad \forall w \in W \quad (6)$$

$$\sum_a \sum_p b_{ap} \cdot \beta_{ap} + \sum_a \sum_r b_{ar} \cdot \gamma_{ar}(\omega) + \sum_a \sum_r \sum_p (b_{ar}^p - b_{ar}) \cdot \lambda_{pr} \cdot \beta_{ap} \cdot \gamma_{ar}(\omega) \leq B, \quad (7)$$

$$\sum_{w \in W} \sum_{k \in K_w} \delta_{ak} \cdot f_k^w(\omega) \leq c_a(\omega) + \sum_r \Delta c_{ap} \cdot \beta_{ap} + \sum_r \Delta c_{ar}(\omega) \cdot \gamma_{ar}(\omega), \quad \forall a \in A \quad (8)$$

$$\sum_{a \in k} t_a(\omega) + \sum_{a \in k} \sum_r (t_{ar} - t_a(\omega)) \cdot \gamma_{ar}(\omega) + Q_k^w(\omega) \leq T_w^{\max} + M \cdot (1 - y_k^w(\omega)), \quad \forall k \in K_w, w \in W \quad (9)$$

$$f_k^w(\omega) \leq M y_k^w(\omega), \quad \forall k \in K_w, w \in W \quad (10)$$

$$Q_k^w(\omega) - q_{ar} \cdot \gamma_{ar}(\omega) - \sum_p (q_{ar}^p - q_{ar}) \cdot \lambda_{pr} \cdot \beta_{ap} \cdot \gamma_{ar}(\omega) \geq 0, \quad \forall a \in k, k \in K_w, w \in W \quad (11)$$

$$\sum_r \gamma_{ar}(\omega) \leq 1, \forall a \in A, r \in R \quad (12)$$

$$\gamma_{ar}(\omega) \in \{0, 1\} \quad \forall a \in A, r \in R \quad (13)$$

$$y_k^w(\omega) \in \{0, 1\}, f_k^w(\omega) \text{ integer}, \quad \forall k \in K_w, w \in W \quad (14)$$

The objective function (2) in the first stage seeks to maximize the expectation of  $Z(\omega)$  over disruption realizations  $\omega$  for a given decision on preparedness actions, where  $Z(\omega)$  is the maximum total post-disaster number of shipments,  $f_k^w(\omega)$ , that can be made between all O-D pairs for a given disruption scenario  $\omega$ . Thus, objective function (2) gives the maximum expected total throughput. First-stage constraint (3) specifies that at most one set of preparedness actions can be taken for each link and (4) restricts the preparedness variable  $\beta_{ap}$  to be binary. Demand constraints (6) guarantee that the total number of shipments pushed along all paths between a particular O-D pair  $w$  will not exceed the original pre-disaster demand for the O-D pair. Constraint (7) requires that the total cost of all chosen



pre-disaster preparedness action and post-disaster recovery actions not exceed available budget  $B$ . The monetary interaction between pre-disaster preparedness and post-disaster recovery actions is accounted for by preparedness-recovery action relationship matrix  $\lambda$ . This matrix consists of predetermined binary elements that specify whether the preparedness action  $p$  impacts recovery action  $r$  in terms of its implementation cost. That is, the cost of implementing recovery action  $r$  on a link  $a$ ,  $b_{ar}^p$ , given that a relevant preparedness action  $p$  is taken pre-disaster, i.e.  $\lambda_{pr} = 1$ , can decrease the implementation cost of recovery activity  $r$  when taken alone. Note that constraint (7) is nonlinear.

Constraints (8) account for the impact of preparedness actions on network capacity enhancement. An augmented capacity  $\Delta c_{ap}$  can be achieved by taking preparedness action  $p$  in link  $a$ . The post-disaster capacity of the link, thus, is the sum of the post-disaster reduced capacity for the given disruption scenario and the augmentation in capacity obtained by implementing preparedness and recovery actions. Note that  $\Delta c_{ar}$  is a function of the disaster scenario realization. This permits the modeling of situations where the impact of a recovery action may be minimal under certain scenarios. For example, pumping water from a roadway link will provide added capacity in a flooding scenario, but will provide little aid in mitigating the effects of an earthquake.

The fact that taking preparedness actions in advance can reduce the implementation time of recovery actions is taken into account through level-of-service (LOS) constraints (9) through (11). These constraints limit the total traversal time, including link travel time and recovery action implementation time, between each O-D pair  $w$  for a pre-defined threshold  $T_{\max}^w$ . This restriction holds only for paths along which flow is ultimately sent.  $y_k^w(\omega)$  specifies whether path  $k$  is used for sending shipments between O-D pair  $w$ . If it is not, the limitation is not imposed. If for a given shipment the total traversal time exceeds the

threshold, the shipment is considered unserved and is not included in the throughput computation. The implementation time of recovery actions depend on whether or not certain preparedness actions have been taken, as described in constraints (11). Recovery action implementation times are accounted for in LOS computations.

At most one set of recovery actions can be taken along each link as specified through constraints (12). Constraints (13) restricts recovery action variables,  $\gamma_{ar}$ , to be binary and constraints (14) impose integrality and non-negativity restrictions for second stage variables.

Note that for simplicity of notation, as written, all preparedness and recovery actions are presumed to be available for every link. This need not be the case.

## CHAPTER 4: SOLUTION METHODOLOGY

### 4.1 OVERVIEW OF SOLUTION METHODOLOGY

The aim of the solution methodology is to determine the optimal portion of the budget to spend on preparedness and amount of the budget to save for post-disaster recovery given future network states that could result from one of many possible disaster scenarios. The probability of each disaster scenario is assumed to be known *a priori* and it is possible that no such disaster scenario will be realized. The optimal investment plan will result in the maximum expected resilience index for the network.

(*RPO*) is a two-stage stochastic program with binary first-stage decision variables and binary, as well as integer, second stage decision variables. The L-shaped method of Van and Wets (1969), a variant of Benders' decomposition, is typically applied in solving two-stage stochastic programs. Within this approach, a single variable,  $\theta$ , is used to approximate the expected value of the second-stage recourse function. The technique seeks the solution corresponding with the optimal  $\theta$ , and  $\theta$  is determined iteratively by using LP duality to construct a convex piecewise linear approximation of the objective function. Because this approach requires that dual variables be obtained in each iteration, it cannot be applied in solving stochastic programs with integer decision variables. Instead, an integer L-shaped method for problems with binary first-stage variables and arbitrary second stage variables developed by Laporte and Louveaux (1993) and applied successfully to a vehicle routing problem (Laporte and Louveaux, 1998) is employed. Laporte and Louveaux's technique extends earlier work by Wollmer (1980) for two-stage stochastic programs with binary first-stage and continuous second-stage decision variables.

Like the standard L-shaped method, the integer L-shaped method applied herein

begins with the decomposition of the two-stage stochastic program into a master problem (MP) in which integrality constraints are relaxed and a set of subproblems (SPs), one subproblem for each network state. A branch-and-bound tree structure is imposed. Nodes, referred to as pendant nodes, are added to the tree during the procedure. The initial problem, in which no variables are fixed, is solved at the root of the tree.

At any step  $v$ , solution of the master problem results in an approximation of  $E_{\tilde{\omega}}[Z(\omega)]$  (from first stage objective function (2)), denoted  $\theta^v$ . If solution of the master problem is not integer, two new branches are created from the current node to two new pendant nodes, fixing the value of a chosen variable, and the process continues by branching.

If the solution is integer, decision variables from the master problem are fixed within the subproblems and the subproblems are solved. An expectation, which in step  $v$  is denoted as  $\psi(\beta^v)$ ,  $\beta^v = \{\beta_{ap}^v\}_{a \in A, p \in P}$ , is taken over the resulting subproblem objective function values weighted by network state probabilities. If  $\psi(\beta^v)$  is no less than  $\theta^v$ , an optimality cut is generated from the solutions of the subproblems and an absolute lower bound associated with the subproblems. This cut is added back to the master problem starting the next step  $v+1$ . The master problem is resolved. Otherwise, if  $\psi(\beta^v) < \theta^v$ , the current node is fathomed and the process continues at the next pendant node.

When no pendant nodes of the tree that have not yet been considered remain, the entire procedure terminates. As optimality cuts are added to the master problem, the master problem becomes increasingly constrained. This integer L-shaped algorithm is guaranteed to converge in a finite number of steps.

To generate network states with properties related to a chosen set of scenario classes (e.g. earthquake, flooding,...), Monte Carlo simulation is employed. Through Monte Carlo simulation, link capacities for a set of network states are set through repeated sampling so as

to approximate pre-specified probability distribution functions and to preserve a given correlation structure among the random variables. The greater the number of samples (i.e. network states), the more accurate the approximation. The approach developed by Chang et al. (1994) and employed by Chen and Miller-Hooks (forthcoming) in a similar context is applied herein to generate multivariate correlated random variates of arc capacities. These sample network states are generated during initialization. The general framework for the integer L-shaped method employed herein is illustrated in Figure 2.

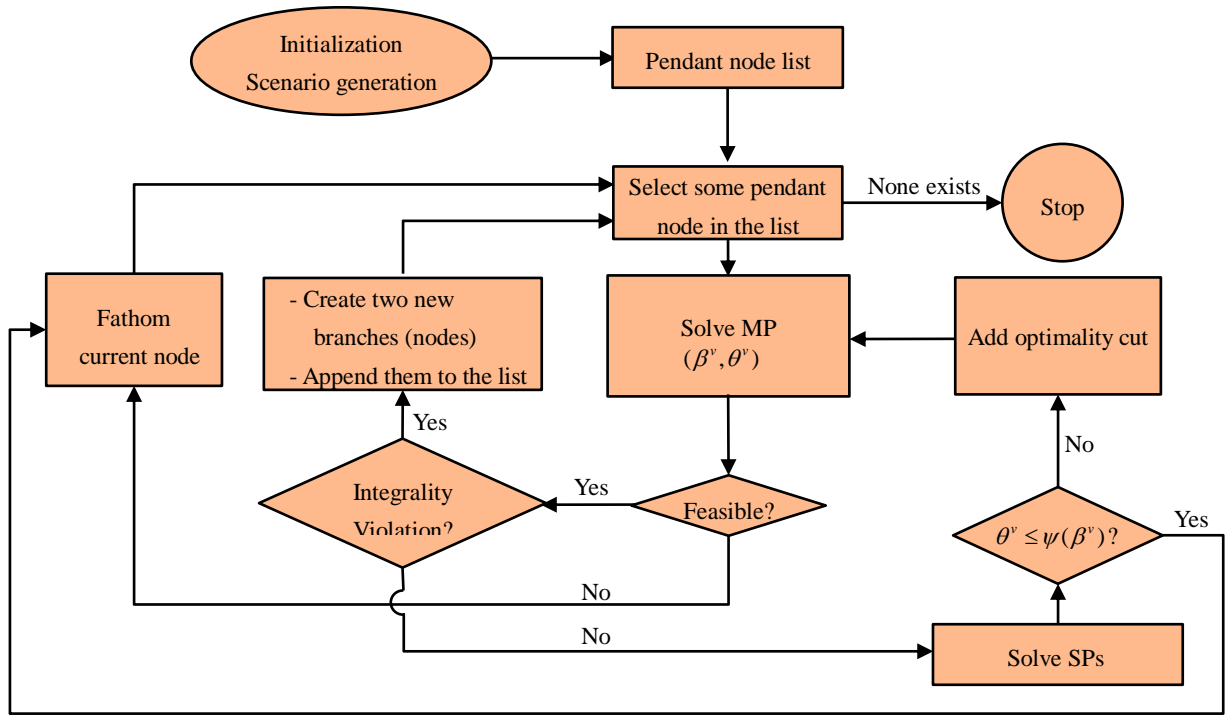


Figure 2. Flowchart for integer L-shaped method

## 4.2 APPLYING THE INTEGER L-SHAPED METHOD FOR SOLUTION OF (RPO)

To implement the integer L-shaped method for solution of the (RPO), the problem is treated as one of minimization, i.e. (2) is replaced by (15), and the problem is decomposed into a master problem (MP) and subproblems (SPs) as follows.

$$\min E_{\tilde{\omega}} \left[ \min \left\{ - \sum_{w \in W} \sum_{k \in K_w} f_k^w(\omega) \right\} \right] \quad (15)$$

$$(MP) \quad \min \theta \quad (16)$$

$$s.t. \quad \sum_p \beta_{ap} \leq 1 \quad (3)$$

$$\sum_a \sum_p b_{ap} \beta_{ap} \leq B, \quad (17)$$

$$f(\theta, \beta) \geq 0, 0 \leq \beta_{ap} \leq 1, \quad (18)$$

where  $\theta$  is the approximation of expected second-stage objective function value,  $\beta = \{\beta_{ap}\}_{a \in A, p \in P}$ , and  $f(\theta, \beta) \geq 0$  is the set of linear optimality cuts generated during the algorithm. A valid constraint (17) that requires that the cost of preparedness actions not exceed the total budget is also added to the master problem.

$$(SPs) \quad E_{\tilde{\omega}} \left[ \min \left( - \sum_{w \in W} \sum_{k \in K_w} f_k^w(\omega) \right) \right] \quad (19)$$

$s.t.$  constraints (6 ~ 14).

For  $|P|$  available preparedness activities and  $|A|$  arcs, the number of elements in  $\beta = |P| \cdot |A|$ , denoted here by  $n$ . Let the  $i$ th element in vector  $\beta$  be given as  $\beta_{ap}(i)$ . Then,  $\{\beta_{ap}(1), \beta_{ap}(2), \dots, \beta_{ap}(n)\} \subset \{0, 1\}^n$ . Index set  $I = \{i : \beta_{ap}(i) = 1\}$ . Optimality cuts are obtained through equation (20).

$$\theta \geq [\psi(\beta) - L] \left[ \sum_{i \in I} \beta_{ap}(i) - \sum_{i \notin I} \beta_{ap}(i) \right] - [\psi(\beta) - L](|I| - 1) + L, \quad (20)$$

where  $L$  is a finite absolute lower bound on (19). Since the total throughput cannot exceed the total pre-disaster demand, we can set the negative of the total demand as a valid absolute lower bound. Thus,  $L = -\sum_w D_w$ . The validity of the optimality cuts (20) is a consequence of the fact that  $\sum_{i \in I} \beta_{ap}(i) - \sum_{i \notin I} \beta_{ap}(i) \leq |I|$ .

In implementing the procedure, branching is based on the most fractional variable.

Addition of the optimality cuts to the master problem is implemented following the scheme proposed by Listes (2004), which was shown to reduce computation times. Unlike the original implementation of the integer L-shaped method in which any cuts added to the master problem are applied until termination, this scheme employs a dynamic list of optimality cuts. Optimality cuts generated at an ancestor node in the branch-and-bound tree are imposed on only descendant nodes in the tree.

Constraint (7) includes the nonlinear term  $\beta_{ap} \cdot \gamma_{ar}(\omega)$ . Since the elements of  $\beta$  are set in the first stage, they can be treated as constants in the second stage where the constraint is enforced. Thus, the structure of the solution approach is exploited to eliminate concerns with nonlinearities in the formulation.

## **CHAPTER 5: ILLUSTRATIVE CASE STUDY**

To assess the impact of preparedness on resilience level, the integer L-shaped method was applied on the Double-Stack Container Network introduced by (Morlok and Chang, 2004; Sun et al., 2006) and considered in (Chen and Miller-Hooks, forthcoming). The solution methodology was implemented in C++ and run in the Microsoft Visual Studio C++ 2005 environment, employing ILOG's CPLEX 10.1 and the Concert Library. The computations were carried out on a personal workstation with a Pentium 4 3.20 GHz processor with 2.00 GB RAM running Windows XP Professional Edition.

### **5.1 ILLUSTRATION ON THE DOUBLE-STACK CONTAINER NETWORK**

The Double-Stack Container Network depicted in Figure 3 provides a simplified representation of the intermodal freight network in the Western U.S. It contains 8 nodes, representing major cities, 24 rail one-way links and 22 bi-directional virtual highway links. It is assumed that highway links have sufficient capacity to support all freight transport demand for the region. Travel time estimates for the virtual highway links were obtained from Google Maps. Intermodal links exist at every node (i.e. at every city), connecting each rail terminal with the highway network. 17 O-D pairs are considered.



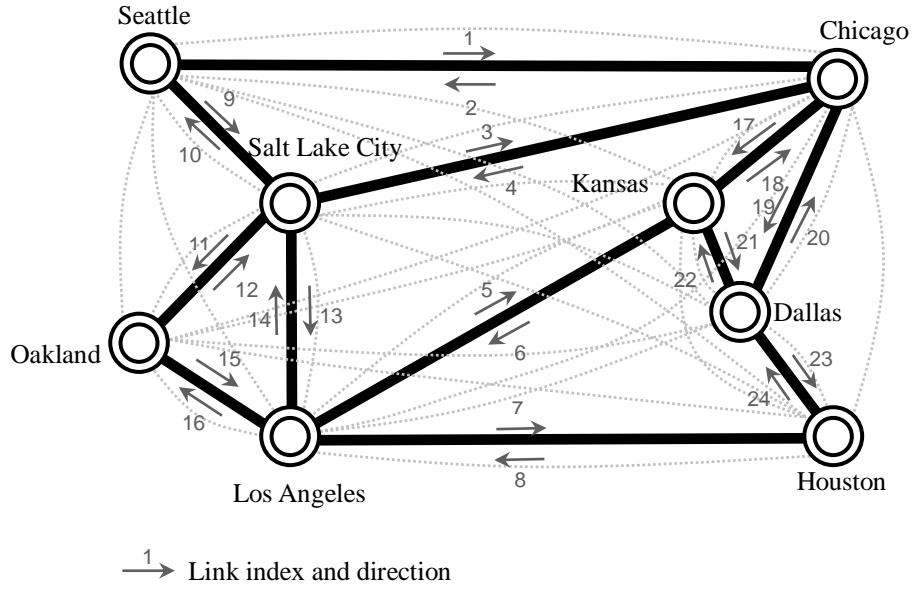


Figure 3. Western U.S. Double-Stack Container Network

## 5.2 DISASTER SCENARIOS, PREPAREDNESS AND RECOVERY ACTIVITIES

Consistent with (Chen and Miller-Hooks, forthcoming), five scenario classifications designed for replicating five general disaster event types (bombing (1), terrorist attack (2), flood (3), earthquake (4) and intermodal terminal attack (5)) were considered in the experimental runs. Each scenario realization corresponds with a setting of the random arc attributes. A bombing scenario was replicated by reducing link capacities on randomly selected arcs within the network. In the case of a terrorist attack, links were assumed to incur significant damage, and thus, significant reduction in capacity. Moreover, the impact of the attack is greatest closest to the attack scene and diminishes with distance. For the flood scenario, the capacities of multiple connected links were assumed to be zero. To replicate an earthquake, links were randomly selected over a large area and their capacities were randomly reduced. In the last disaster event type, rail service is assumed to be inoperable into and out of the terminals in

Chicago and Los Angeles and all cargo.

Monte Carlo simulation was used to generate each network state. Network attributes for each state are characterized through correlation matrices. The arc capacities are assumed to be uniformly distributed and travel times increase in proportion to capacity decrease. For one unit decrease in capacity, an increase of 10% is incurred in travel time. Since a virtual highway network is employed, and alternative roadway paths may exist that connect the same O-D pair, it is assumed that only rail links will be impacted by a disaster.

As the computational effort required to solve a stochastic program is significant, and that effort increases linearly with each realization, only 100 realizations from each disaster classification, or a total of 500 realizations, were generated in this experiment. A larger number of realizations will reduce the sampling error. For simplicity, we assume the probability of each of the five disaster scenarios to equal 0.2.

One might also consider a case in which the probability of any disaster arising is quite small, thus adding an additional scenario in which no changes to the network occur. Because the budget is fixed, there will be no change in solution as a result of considering a no disaster scenario when disaster scenarios are considered proportionately identical in both cases. If an objective of minimizing expenses were considered, however, solution in the case involving a positive probability of no disaster will result in reduced spending in the preparedness stage and greater spending in the response stage.

To compare the resilience level when preparedness activities are implemented, the same six recovery activities defined by Chen and Miller-Hooks (forthcoming) are available. The duration time, cost of implementation, impact to the link capacity, and candidate link for application of each recovery activity are listed in Table 1.

Table 1. Characteristics of recovery activities

Recovery activities	Recovery activity duration (units)	Cost (units)	(% increase in link capacity)	Candidate links
R1	2	6	10	1-12
R2	1	10	10	1-6
R3	6	1	5	7-12
R4	4	4	10	1,3,5,7,9,11
R5	3	8	15	2,4,6,8,10,12
R6	3	10	Return to original capacity	1-12

Two preparedness options that might be taken in preparation for a disaster event are available for implementation in this study: special training of personnel along specific routes to enhance recovery (P1) and prepositioning of water pumps (P2). These actions are coupled with a level of retrofit that provides additional enhancement to the capacities of links to which these actions are applied. Additional information pertaining to the preparedness actions are given in Table 2.

Table 2. Characteristics of preparedness activities

Preparedness activities	Cost (units)	(% increase in link capacity)	Candidate link	Applicable for disaster scenarios
P1	3	10	1-6	1-5
P2	2	8	1-6	3

The implementation time and cost of all six recovery actions on a given link will be reduced if P1 is taken on that link. It is assumed that the reduction is 20% for both travel time and cost. The benefits of P2 will only be realized if recovery action 4 (e.g. pumping water) is implemented. If P2 is chosen for application between two cities, its benefits will be received in both directions. The preparedness-recovery relation matrix is shown in Table 3.

Table 3. Preparedness-recovery activity relationship matrix,  $\lambda$

	<b>R1</b>	<b>R2</b>	<b>R3</b>	<b>R4</b>	<b>R5</b>	<b>R6</b>
P1	1	1	1	1	1	1
P2	0	1	1	0	0	0

A budget of 30 units and an unlimited budget are considered. The maximum allowable travel time for each O-D pair is assumed to be 1.5 times the travel time of the shortest path between the O-D pair in the original network. That is, an increase of as much as 50% in travel time is assumed to produce an acceptable level of service given the disaster occurrence.

### 5.3 EXPERIMENTAL RESULTS

Results of the numerical experiments in terms of obtained resilience level are given in Figure 4. Because equal probability of each type of scenario was presumed, one can obtain the total resilience for the network over all scenario classes by simply adding the conditional resilience values and dividing by five. To judge the impact of preparedness actions on the network's resilience level, three additional runs were completed. The runs are synopsisized in Table 4, along with overall resilience level (i.e. expected throughput over all scenarios). Results are shown for each disaster category. The first of the additional runs considers the case where no recovery or preparedness activities are available, a measure comparable to some notions of reliability. The second set of additional runs permits only preparedness actions, while the third considers only recovery options. Results from these additional runs are provided for comparison in Figure 4.

Table 4. Description of implementations of resilience measure

Run	Description	Overall Resilience, $\alpha$
1	no recovery and preparedness activities taken	0.54
2	only preparedness activities are implemented	0.65
3	only recovery activities are implemented	0.68
4	both preparedness and recovery activities can be undertaken	0.72

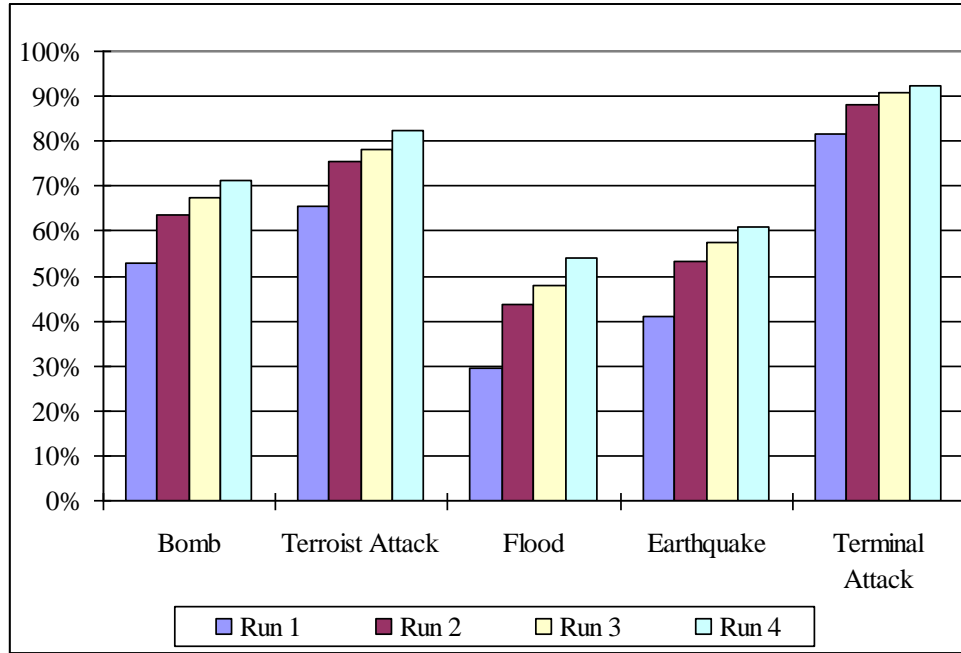


Figure 4. Resilience level by disaster scenario (budget=30)

The results indicate that both preparedness and recovery activities can significantly improve network resilience level. When taken alone, the recovery actions have greater impact than the preparedness actions. Moreover, the implementation of both preparedness and recovery activities provides a higher resilience level than either can reach alone. As evidence of this, note that on their own recovery activities led to a 14% improvement in the overall resilience level, while just under an 11% increase was obtained through preparedness activities alone. An 18% increase was obtained when both types of activities were available.

The average portion of the budget allocated to preparedness and recovery activities, as

well as the maximum and minimum cost incurred over the set of disaster scenarios, are provided in Table 5.

Table 5. Cost of activities (budget=30)

Scenario	Runs							
	Run 1		Run 2		Run 3		Run 4	
	CP	CR	CP	CR	CP	CR	CP	CR
Bomb	0	0	18	0	0	26.7	16	12.6
Terrorist	0	0	18	0	0	28.2	16	12.5
Attack	0	0	18	0	0	29.6	16	13.4
Flood	0	0	18	0	0	29.3	16	13.3
Earthquake	0	0	18	0	0	26.6	16	11.8
Terminal	0	0	18	0	0			
Attack	0	0	18	0	0			
M	0		18		30		30	
m	0		18		10		18	

The budget was not restrictive in Run 2. That not all the budget is used in each scenario may be a function of the discrete nature of activity costs. Results of Run 4 indicate that more funds were spent on preparedness actions than on recovery actions in the optimal solution. If a scenario involving no disaster were considered and this scenario were given a relatively high probability, one would expect that much more of the budget would be spent on recovery actions than on preparedness. It may also be particular to this example.

To further investigate the allocation of the budget between preparedness and recovery options, Runs 3 and 4 were repeated with an unlimited budget. Runs 1 and 2 need not be reconsidered since the maximum budget used in any realization was less than the original budget permitted. Results from these runs are provided in Figure 5. The associated allocation of the budget across activity types is given in Table 6.

Table 6. Cost of activities (unlimited budget)

Scenario	Runs			
	Run 3		Run 4	
	CP	CR	CP	CR
Bomb	0	38.7	18	27.9
Terrorist Attack	0	41.9	18	32.6
Flood	0	62.6	18	48.1
Earthquake	0	47.5	18	33.8
Terminal Attack	0	40.4	18	30
Maximum Cost	76		79	
Minimum cost	12		16	

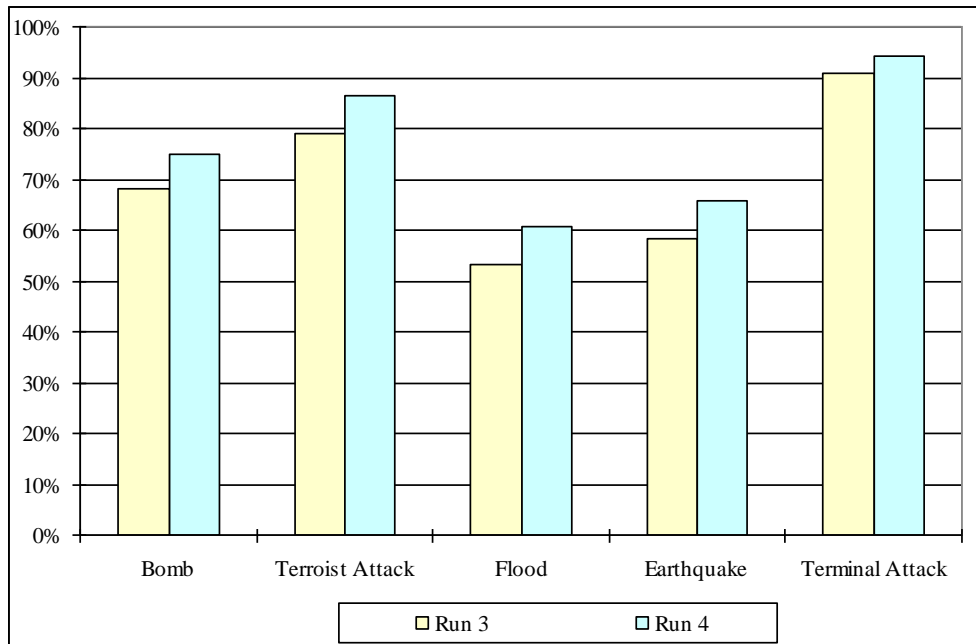


Figure 5. Resilience level by disaster scenario (unlimited budget)

With increased budget, the resilience level for Runs 3 and 4 are 0.70 and 0.76, respectively, leading to an increase in resilience by 2% and 4%, respectively. The maximum cost of actions taken in any disaster realization was 79 units. It was found that for many disaster realizations, no additional improvement in resilience could be obtained from increasing the budget.





## CHAPTER 6: CONCLUSIONS

This paper revisits the notion of resilience proposed by Chen and Miller Hooks (forthcoming), which accounts for recovery actions that can be taken post-disaster within a limited timeframe and budget. Herein, this notion is extended to include preparedness actions that can provide increased recovery capability, in addition to increased coping capacity. The concept is applied in the context of an intermodal rail application, but its relevance extends beyond transportation. The inclusion of preparedness decisions in determining a network's resilience level provides an extra level of decision support; however, this addition increases the problem difficulty. Whereas omission of preparedness options permits the problem's decomposition into a set of deterministic subproblems, its inclusion prevents it. In this paper, the problem of measuring a network's resilience level and determining the optimal set of preparedness and recovery actions needed to achieve this level given budget and level of service constraints is formulated as a two-stage stochastic program. An integer L-shaped method accredited to Laporte and Louveaux (1993) is proposed for its solution. The solution method decomposes the problem into a master problem and set of subproblems, each associated with a different disaster realization. Monte Carlo simulation is employed for the generation of the disaster realizations. This decomposition eliminates concerns associated with nonlinearities in the budget constraint of the formulation. The solution approach was applied on the Double-Stack Container network abstracting the intermodal rail system of the Western U.S. Optimal allocation of a limited budget between preparedness and recovery options is studied and the proposed expanded notion of resilience is compared with resilience without optimizing preparatory actions and a comparable notion of reliability.

Results of the numerical experiments provide some insight into optimal investment

allocation of a fixed budget between preparedness and recovery stages. While improvements in resilience level are obtained from taking preparedness or recovery actions alone, the highest resilience level is attained when both preparedness and recovery options are available. In general, whether the budget or the available preparedness and recovery options were the limiting factors, greater benefit was derived through greater allocation of funds to the preparatory actions. If a scenario involving no disaster were introduced; however, one should expect that a greater portion of the budget would be reserved for the recovery stage.

The techniques presented herein will substantively increase our ability to aid in pre-disruption network vulnerability assessment and making pre-disaster vulnerability-reduction investment decisions. Quick identification of the appropriate actions to take can play a crucial role in lessening post-disaster economic and societal loss. Competing measures, such as reliability and flexibility, that do not consider quick and inexpensive recovery actions that may be taken post-disaster may underestimate the network's ability to cope with unexpected events and may lead to unnecessary or misdirected investment.

The problem instance studied within this paper is rather small. To represent a real-world transport network with greater fidelity, the problem size will grow substantially. Exact solution of such large problem instances will be difficult to obtain. The conceptualization of the problem as a two-stage stochastic program and suggested solution approach provide a methodology for solving small benchmark problems against which the performance of a developed heuristic can be measured.

Alternatives to the (RPO) formulation might be considered. For example, one might envision a version of the problem with an objective of minimizing the required budget needed to obtain a specific resilience level. Other formulations might incorporate details for

alternative applications. The authors are currently conceiving of a similar notion of resilience for passenger traffic, where no load can be left unserved.

Also discussed in (Liu et al., 2009), added realism may be obtained by considering decision-dependent network state probabilities (addressed by Jonsbraten et al. (1998) in the context of generic discrete decision problems through an implicit enumeration method). This is because the probability of a specific realization of a disaster scenario may be impacted by the taking of a preparedness action. That is, if a component of the infrastructure is hardened, the probability that its capacity would be substantially reduced may be decreased as a result of the action. Such consideration, however, greatly reduces the problem's tractability.

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